**Write R Scripts or use R to perform any mathematical operations while solving the following problems.**

**Problem 1: Faulty brakes in Delivery trucks**

A delivery company has 12 trucks, of which 4 have faulty brakes. If an inspector randomly chooses two of the trucks for brake check, what is the probability that neither one has faulty brakes?

**[Ans:]** Prob(neither faulty) = C(8,2)/C(12,2) = 14/33 = 0.4242

**Problem 2: Conditional Probability**

A total of 500 married working couples were polled about their annual salaries, with the following information resulting. 212 couples reported both the husband and wife earned less than $25,000, 198 couples reported only the husband earned more than $25,000, 36 couples reported that only the wife earned more than $25,000, and 54 couples reported that both the husband and wife earned more than $25,000. If one of the couples is randomly chosen, what is

a. the probability that the husband earns less than $25,000?

b. the conditional probability that the wife earns more than $25,000 given that the husband earns more than this amount?

c. the conditional probability that the wife earns more than $25,000 given that the husband earns less than this amount?

**[Ans:]**

|  |  |  |
| --- | --- | --- |
|  | **Wife < 25000** | **Wife > 25000** |
| **Husband < 25000** | 212 | 36 |
| **Husband > 25000** | 198 | 54 |

1. P(Husband < 25000) = (212+36)/500 = 248/500 = 0.496
2. P(Wife > 25k|Husband > 25k) = 54 / (198+54) = 54/252
3. P(Wife > 25k|Husband < 25k) = 36 / (212+36) = 36/248

**Problem 3: Chain Rule**

A recent college graduate is planning to take the first three data science examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she with take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the conditional probability that she passes the second one is 0.8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is 0.7. What is the probability that she passes all three exams?

**[Ans:]**

P(pass 1st exam) = 0.9

P(pass 2nd exam | passed 1st exam) = 0.8

P(pass 3rd exam | passed 1st and 2nd exams) = 0.7

So, P(Pass all 3 exams) = P(pass 1st exam)\*P(pass 2nd exam | passed 1st exam) \* P(pass 3rd exam | passed 1st and 2nd exams) = 0.9 \* 0.8 \* 0.7 = 0.504

**Problem 4: Bayes Rule**

a. Two dice are tossed, one green and one red. What is the conditional probability that the number on the green die is 6, given that the sum on the two dice is 7?

b. Suppose that 5% of men and 0:25% of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.

**[Ans:]**

a. Using Bayes theorem, we have

P(Green=6|Sum=7) = P(Green=6 and Sum=7) / P(Sum=7)

P(Green=6 and Sum=7) = P(Green=6 and Red=1) = 1/6\*1/6 = 1/36

P(Sum=7) = 6/36

So, P(Green=6|Sum=7) = (1/36)/(6/36) = 1/6

Without using Bayes theorem, P(Green=6|Sum=7) = 1/6 since Green=6 is only one of the six possibilities when the Green+Red = 7, viz, (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)

b. Without using Bayes theorem, let us assume the following hypothetical data:

|  |  |  |
| --- | --- | --- |
|  | Color blind | Not color blind |
| Male | 20 | 380 |
| Female | 1 | 399 |

So, P(Male|Color blind) = 20/21.

Using Bayes theorem:

P(cb|male) = 0.05

P(cb|female) = 0.0025

P(male) = P(female) = 0.5

Need to find P(male|cb).

P(male|cb) = P(male and cb) / P(cb)

= P(male and cb) / [P(cb and male) + P(cb and female)]

P(male and cb) = P(cb and male)

= P(cb|male)\*P(male)

= 0.05\*0.5

P(cb and female) = P(cb|female)\*P(female) = 0.0025 \* 0.5

So, P(male|cb) = (0.05\*0.5) / [(0.05\*0.5) + (0.0025\*0.5)]

= 0.05/0.0525

= 5/5.25

= 20/21

**Problem 5: Rain forecast for Marie’s Marriage**

Marie is getting married tomorrow at an outdoor ceremony in the desert and the weatherman is trying to predict whether it will rain tomorrow or not. In recent years, it has rained only 5 days each year (5/365 = 0.014). When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. Unfortunately, the weatherman has predicted rain for tomorrow.

a. What is the probability that it will rain on Marie's wedding?

b. What is the best hypothesis weatherman can conclude?

**[Ans:]**

a. Without using Bayes theorem, this is the data table for 365 days:

|  |  |  |
| --- | --- | --- |
|  | Rain forecasted | ‘No rain’ forecasted |
| Rain | 4.5 {90% of 5 days} | 0.5 |
| No rain | 36 {10% of 360 days} | 324 |

So, P(rain|rain forecast) = 4.5/(4.5+36) = 4.5/40.5 = 1/9

**Using Bayes theorem:**

P(rain) = 5/365

P(no rain) = 1 – P(rain) = 360/365

P(rain forecast|rain) = 0.9

P(rain forecast|no rain) = 0.1

P(rain|rain forecast) = P(rain forecast|rain)\*P(rain) / P(rain forecast)

P(rain forecast) = P(rain forecast and rain) + P(rain forecast and ‘no rain’)

P(rain forecast and rain) = P(rain forecast|rain)\*P(rain)

P(rain forecast and ‘no rain’) = P(rain forecast|no rain)\*P(no rain)

So, P(rain|rain forecast) = (0.9 \* 5/365) / [(0.9 \* 5/365) + (0.1 \* 360/365)]

= (0.9 \* 5) / [(0.9 \* 5) + (0.1 \* 360)]

= 4.5 / 40.5

= 1/9

b. Even when the weatherman predicts rain, it rains only about 1/9th of the time. Despite the weatherman’s gloomy prediction, there is a good chance that Marie will not get rained on her wedding.

**Problem 6: Gambling Insight**

In casino, a roulette wheel has the numbers 1 through 36, as well as 0 and 00. If you bet $1 that an odd number comes up, you win or lose $1 according to whether or not that event occurs.

a. Find the expected gain/loss for casino for each game?

b. What are the expected gain/loss for casino if stakes of bet are increased from $1 to $10? What will be total expected gain/loss for 10000 games after the increased stake?

c. What is the standard deviation and how do you interpret it?

**[Ans:]**

1. 0 and 00 are considered even. So, there are 20 even numbers and 18 odd numbers.

P1 = P(win) = P(odd) = 18/38

P2 = P(loss) = 20/38

X1 = Winning amount = 1

X2 = Losing amount = -1

Expected gain/loss = E(X) = P1X1 + P2X2 = 18/38\*(1) + (20/38)\*(-1)

= -1/19

So, we expect to lose $1/19 or 5.26 cents

1. If stakes were increased to $10, X1 = 10 and X2 = -10.

So, E(X) = -10/19 = -52.6 cents

Total E(X) for 10000 games = -(10/19)\*10000 = -$5263.16

1. Variance = P1\*[X1 – E(X)]^2 + P2\*[X2 – E(X)]^2

= (18/38)\*(1+1/19)^2 + (20/38)\*(-1+1/19)^2

= [(18\*40\*40) + (20\*36\*36)] / (38\*38\*38)

= 0.9972299

Standard deviation = sqrt(Variance) = 0.998614

For discrete random variables, standard deviation is a measure of how much the values differ from the expected value.

**Problem 7: Gold Investment**

You have $10000, and a commodity like gold presently sells for $20 per gram. Suppose that after one week the commodity will sell for either $10 or $40 per gram, with these two possibilities being equally likely. A trading strategy is a plan specifying amounts of the commodity to buy or sell at the beginning and end of the week.

a. If your objective is to maximize the expected amount of money that you possess at the end of the next week, what strategy should you employ?

b. If you objective is to maximize the expected amount of the gold that you possess at the end of the week, what strategy should you employ?

**[Ans:]** At the beginning of the week, let us invest $x in gold. So, we have cash = $(10000-x) and gold = x/20 grams.

*Scenario 1*: Gold price dropped to $10/gram.

To maximize gold, buy gold with the remaining money. So, gold = x/20 + (10000-x)/10 = (1000 – x/20) grams

To maximize money, sell all the gold. So, money = (10000-x) + (x/20\*10) = $(10000 – x/2)

*Scenario 2*: Gold price increased to $40/gram.

To maximize gold, buy gold with the remaining money. So, gold = x/20 + (10000-x)/40 = (250 + x/40) grams

To maximize money, sell all the gold. So, money = (10000-x) + (x/20\*40) = $(10000 + x)

Since both the scenarios are equally likely, the expected value is their average.

So, we have:

Expected ‘maximum gold’ = (1000 – x/20 + 250 + x/40) /2 = (625 – x/80) grams

Expected ‘maximum money’ = (10000 – x/2 + 10000 + x) / 2 = $(10000 + x/4)

Therefore, to maximize the expected gold, one should NOT invest any money in gold at the beginning, but buy gold with all the money at the end of the week. Then, maximum expected gold = (625 – 0/80) = 625 grams.

Conversely, to maximize the expected money, one should invest ALL the money in gold at the beginning, and sell all the gold at the end of the week. Then, maximum expected money = $(10000 + 10000/4) = $12,500.

**Problem 8: Expected value & Standard deviation**

I. The expected value and variance of a coin toss are?

a.50, .50 b.50, .25 c.25, .50 d.25, .25

**[Ans:]** X1 = 1, X2 = 0, P1 = ½, P2 = ½

E(X) = ½(1) + ½ (0) = 0.5

Variance = ½(1-0.5)^2 + ½(0-0.5)^2 = 0.25

So, answer is b)

II. Suppose that the probability function shown below reflects the possible lifetimes (in months after emergence) for fruit flies:

x 1 2 3 4 5 6

p(x) 0.30 ? 0.20 0.15 0.10 0.05

a. What proportion of fruit flies dies in their second month?

**[Ans:]** p(2) = 1 – [p(1)+p(3)+p(4)+p(5)+p(6)] = 1 - 0.80 = 0.20

b. What is the probability that a fruit fly lives more than four months?

**[Ans:]** p(x>4) = p(5) + p(6) = 0.10 + 0.05 = 0.15

1. What is the mean lifetime for a fruit fly?

**[Ans:]** Mean lifetime = sigma x\_i\*p(x\_i)

= 1\*(0.3) + 2\*(0.2) + 3\*(0.2) + 4\*(0.15) + 5\*(0.10) + 6\*(0.05)

= 2.7

d. What is the standard deviation of fruit fly lifetimes?

**[Ans:]** Variance = sigma p(x\_i) \*((x\_i - mean)^2)

= (0.3\*1.7\*1.7) + (0.2\*0.7\*0.7) + (0.2\*0.3\*0.3)

+ (0.15\*1.3\*1.3) + (0.1\*2.3\*2.3) + (0.05\*3.3\*3.3)

= 2.31

Standard deviation = sqrt(Variance) = sqrt(2.31) = 1.52